Click-Through Rate prediction: TOP-5 solution for the Avazu contest

Dmitry Efimov Petrovac, Montenegro

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Outline

Provided data

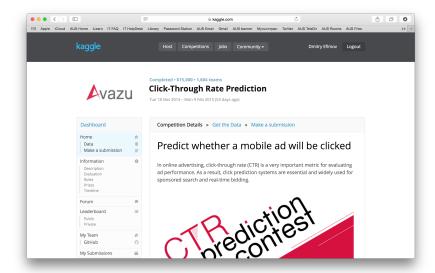
Likelihood features

FTRL-Proximal Batch algorithm

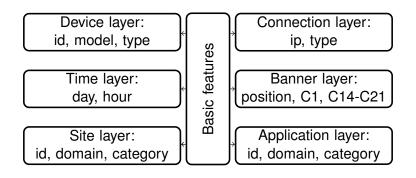
Factorization Machines

Final results

Competition



Provided data



Notations

X: $m \times n$ design matrix

$$m_{train} = 40428967$$

 $m_{test} = 4577464$
 $n = 23$

y: binary target vector of size m $\mathbf{x}^{\mathbf{j}}$: column j of matrix X $\mathbf{x}_{\mathbf{i}}$: row i of matrix X $\sigma(\mathbf{z}) = \frac{1}{1 + \mathbf{e}^{-\mathbf{z}}}$: sigmoid function

Evaluation

Logarithmic loss for $y_i \in \{0, 1\}$:

$$L = -\frac{1}{m} \sum_{i=1}^{m} (y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i))$$

 \hat{y}_i is a prediction for the sample i

Logarithmic loss for $y_i \in \{-1, 1\}$:

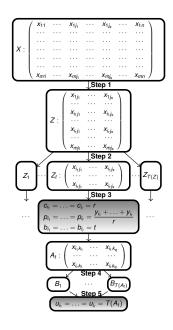
$$L = \frac{1}{m} \sum_{i=1}^{m} \log(1 + e^{-y_i p_i})$$

 p_i is a raw score from the model $\hat{y}_i = \sigma(p_i), \forall i \in \{1, ..., m\}$

Feature engineering

- ▶ **Blocks:** combination of two or more features
- Counts: number of samples for different feature values
- Counts unique: number of different values of one feature for fixed value of another feature
- ▶ **Likelihoods:** $\min_{\theta_t} L$, where $\theta_t = P(y_i \mid x_{ij} = t)$
- Others

Feature engineering (algorithm 1)



function SPLITBYROWS(M)
get partition of the matrix M into $\{M_t\}$ by rows such that each M_t has identical rows

Require:

$$J = \{j_1, \dots, j_s\} \subset \{1, \dots, n\}$$

$$K = \{k_1, \dots, k_q\} \subset \{1, \dots, n\} \setminus J$$

$$Z \leftarrow (x_{ij}) \subset X, i \in \{1, 2, \dots, m\}, j \in J$$

$$\text{for } I = \{i_1, \dots, i_r\} \in \text{SPLITBYROWS}(Z)$$

$$c_{i_1} = \dots = c_{i_r} = r$$

$$p_{i_1} = \dots = p_{i_r} = \frac{y_{i_1} + \dots + y_{i_r}}{r}$$

$$b_{i_1} = \dots = b_{i_r} = t$$

$$A_t = (x_{ik}) \subset X, i \in I, k \in K$$

$$T(A_t) = \text{size}(\text{SPLITBYROWS}(A_t))$$

$$u_{i_1} = \dots = u_{i_r} = T(A_t)$$

Feature engineering (algorithm 2)

function SPLITBYROWS(M) get partition of the matrix M into $\{M_t\}$ by rows such that each M_t has identical rows Require: parameter $\alpha > 0$ $J \leftarrow (j_1, \ldots, j_s) \subset \{1, \ldots, n\}$ increasing sequence $V \leftarrow (v_1, \dots, v_l) \subset \{1, \dots, s\}, v_1 < s$ $f_i \leftarrow \frac{y_1 + \ldots + y_m}{\ldots}, \forall i \in \{1, \ldots, m\}$ for $v \in V$ do $J_{V} = \{j_{1}, \ldots, j_{V}\}, Z = (x_{ij}) \subset X, i \in \{1, 2, \ldots, m\}, j \in J_{V}$ for $I = \{i_1, \dots, i_r\} \in \text{SPLITBYRows}(Z)$ do $c_{i_1} = \ldots = c_{i_r} = r$

$$p_{i_1} = \ldots = p_{i_r} = \frac{y_{i_1} + \ldots + y_{i_r}}{r}$$

 $w = \sigma(-c + \alpha)$ - weight vector

$$f_i = (1 - w_i) \cdot f_i + w_i \cdot p_i, \forall i \in \{1, \dots, m\}$$

FTRL-Proximal model

Weight updates:

$$\begin{aligned} w_{i+1} &= \arg\min_{w} \left(\sum_{r=1}^{i} g_r \cdot w + \frac{1}{2} \sum_{r=1}^{i} \tau_r ||w - w_r||_2^2 + \lambda_1 ||w||_1 \right) = \\ &= \arg\min_{w} \left(w \cdot \sum_{r=1}^{i} (g_r - \tau_r w_r) + \frac{1}{2} ||w||_2^2 \sum_{r=1}^{i} \tau_r + \lambda_1 ||w||_1 + \text{const} \right), \end{aligned}$$

$$\text{where } \sum_{r=1}^{i} \tau_{rj} = \frac{\beta + \sqrt{\sum\limits_{r=1}^{i} \left(g_{rj}\right)^2}}{\alpha} + \lambda_2, \ j \in \{1, \dots, N\},$$

$$\lambda_1, \lambda_2, \alpha, \beta$$
 - parameters, $\tau_r = (\tau_{r1}, \dots, \tau_{rN})$ - learning rates,

 g_r - gradient vector for the step r

FTRL-Proximal Batch model

```
Require: parameters \alpha, \beta, \lambda_1, \lambda_2
    z_i \leftarrow 0 and n_i \leftarrow 0, \forall j \in \{1, \dots, N\}
    for i = 1 to m do
          receive sample vector x_i and let J = \{j | x_{ij} \neq 0\}
          for j \in J do
                w_j = \begin{cases} 0 \\ -\left(\frac{\beta + \sqrt{n_j}}{\alpha} + \lambda_2\right)^{-1} \left(z_j - \text{sign}(z_j)\lambda_1\right) \text{ otherwise} \end{cases}
          predict \hat{y}_i = \sigma(x_i \cdot w) using the w_i and observe y_i
          if y_i \in \{0, 1\} then
                 for i \in J do
                       g_i = \hat{y}_i - y_i - gradient direction of loss w.r.t. w_i
                      \tau_{j} = \frac{1}{\alpha} \left( \sqrt{n_{j} + g_{j}^{2}} - \sqrt{n_{j}} \right)

z_{j} = z_{j}^{2} + g_{j} - \tau_{j} w_{j}
                       n_i = n_i + g_i^2
```

Performance

Description	Leaderboard	
Description	score	
dataset is sorted by app id, site id,	0.3844277	
banner pos, count1, day, hour		
dataset is sorted by app domain,	0.3835289	
site domain, count1, day, hour	0.3033209	
dataset is sorted by <i>person, day, hour</i>	0.3844345	
dataset is sorted by day, hour with 1 iteration	0.3871982	
dataset is sorted by day, hour with 2 iterations	0.3880423	

Factorization Machine (FM)

Second-order polynomial regression:

$$\hat{\mathbf{y}} = \sigma \left(\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \mathbf{w}_{jk} \mathbf{x}^{j} \mathbf{x}^{k} \right)$$

Low rank approximation (FM):

$$\hat{\mathbf{y}} = \sigma \left(\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} (\mathbf{v}_{j1}, \dots, \mathbf{v}_{jH}) \cdot (\mathbf{v}_{k1}, \dots, \mathbf{v}_{kH}) \mathbf{x}^{j} \mathbf{x}^{k} \right)$$

H is a number of latent factors

Factorization Machine for categorical dataset

Assign set of latent factors for each pair level-feature:

$$\hat{y}_{i} = \sigma \left(\frac{2}{n} \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} (w_{x_{ij}k1}, \dots, w_{x_{ij}kH}) \cdot (w_{x_{ik}j1}, \dots, w_{x_{ik}jH}) \right)$$

Add regularization term: $L_{reg} = L + \frac{1}{2}\lambda||w||^2$ The gradient direction:

$$g_{x_{ij}kh} = \frac{\partial L_{reg}}{\partial w_{x_{ij}kh}} = -\frac{2}{n} \cdot \frac{y_i e^{-y_i p_i}}{1 + e^{-y_i p_i}} \cdot w_{x_{ik}jh} + \lambda w_{x_{ij}kh}$$

Learning rate schedule: $\tau_{x_{ij}kh} = \tau_{x_{ij}kh} + \left(g_{x_{ij}kh}\right)^{\epsilon}$ Weight update: $w_{x_{ij}kh} = w_{x_{ij}kh} - \alpha \cdot \sqrt{\tau_{x_{ij}kh}} \cdot g_{x_{ij}kh}$

Ensembling

Model	Description	Leaderboard
	detection and description of all alterial	score
ftrlb1	dataset is sorted by app id, site id,	0.3844277
	banner pos, count1, day, hour	0.001.1277
ftrlb2	dataset is sorted by app domain,	0.3835289
	site domain, count1, day, hour	
ftrlb3	dataset is sorted by person, day, hour	0.3844345
fm	factorization machine	0.3818004
ens	$fm^{0.6} \cdot ftrlb1^{0.1} \cdot ftrlb2^{0.2} \cdot ftrlb3^{0.1}$	0.3810447

Final results

Place	Team	Leaderboard score	Difference between the 1st place
1	4 Idiots	0.3791384	_
2	Owen	0.3803652	0.32%
3	Random Walker	0.3806351	0.40%
4	Julian de Wit	0.3810307	0.50%
5	Dmitry Efimov	0.3810447	0.50%
6	Marios and Abhishek	0.3828641	0.98%
7	Jose A. Guerrero	0.3829448	1.00%

Future work

- apply the batching idea to the Factorization Machine algorithm
- find a better sorting for the FTRL-Proximal Batch algorithm
- find an algorithm that can find better sorting without cross-validation procedure

References

- H.Brendan McMahan et al. "Ad click prediction: a view from the trenches." *In KDD*, Chicago, Illinois, USA, August 2013.
- Wei-Sheng Chin et al. "A learning-rate schedule for stochastic gradient methods to matrix factorization." *In PAKDD*, 2015.
- Michael Jahrer et al. "Ensemble of collaborative filtering and feature engineered models for click through rate prediction." *In KDD Cup*, 2012
- Steffen Rendle. "Social network and click-through prediction with factorization machines." *In KDD Cup*, 2012

Thank you! Questions???

Dmitry Efimov defimov@aus.edu

